

Closed strings from decaying D-branes

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ABSTRACT: We compute the emission of closed string radiation from homogeneous rolling tachyons. For an unstable decaying Dp -brane the radiated energy is infinite to leading order for $p \leq 2$ and finite for $p > 2$. The closed string state produced by a decaying brane is closely related to the state produced by D-instantons at a critical Euclidean distance from $t = 0$. In the case of a D0 brane one can cutoff this divergence so that we get a finite energy final state which would be the state that the brane decays into.

KEYWORDS: Tachyon Condensation, D-branes.

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1. Introduction

A Dp -brane in Bosonic string theory is unstable due to the presence of an open string tachyon in its spectrum. If we displace the tachyon away from the maximum it will start rolling down the potential toward the minimum, which is the closed string vacuum [1]. It is of great interest to understand the “real time” behavior of this process [2–20]–[21]. For one thing, exact time-dependent solutions in string theory are hard to come by and studying them should yield important insights into the fundamental structure of the theory. We would also like to understand the final state an unstable D-brane decays into.

Analysis by Sen at the classical level in open string theory has yielded some interesting surprises [3, 6]. It was found that the late time evolution of the tachyon leads to a pressureless fluid, called “tachyon matter” (see however [15]). Tachyon matter therefore seems to be a new and unexpected degree of freedom. Hence it is natural to understand the precise nature of tachyon matter and in particular to take into account the open and closed strings created by the decay process. In [8, 11] quantum creation of open strings

was computed and it was argued that this effect would destabilize the classical solution (see also [12]).

In this paper we are interested in computing the creation of closed string modes from the rolling tachyon process. Radiation produced by the rolling tachyon has been discussed previously in [13, 14] for the massless closed string modes. In addition supergravity solutions with rolling tachyon sources were discussed in [22, 23]. We will extend this to all massive closed string modes. In [19, 7, 14] it was observed that the “rolling tachyon” boundary state contains exponentially increasing couplings to massive closed string modes and it was suggested in [19, 14] that these will lead to pathological behaviour. However we will see that these couplings are not relevant for the creation of *on-shell physical* modes. We find that the emission into on-shell physical closed string modes is finite for each level. When we sum over all levels we find a divergence in the emitted energy for $p \leq 2$ (similar conclusion has also been reached in [24]). We also find a divergence if $p > 2$ and the worldvolume of the brane is compact with a size comparable to the string scale. The $p = 0$ case is particularly interesting since the computation suggests that all the energy of the initial brane is transferred into closed string modes. The $p \geq 3$ case is particularly relevant for applications to brane world cosmological models.

It turns out that the precise closed string states that are emitted from the brane depend on the physical interpretation of the Sen’s solution. We discuss two possible interpretations. The first is the one implicitly advocated in [3–5] where one thinks of a tachyon field coming up from its true minimum to a point close to the local maximum associated to the unstable D-brane and then back to its true minimum. The second interpretation of the solution is more closely related to the Euclidean computations. Here we cut the Euclidean path integral at $t = 0$ and paste it to the Lorentzian path integral. In this way the Euclidean path integral is viewed as a prescription for setting up initial conditions at $t = 0$. The subsequent Lorentzian evolution is the one resulting from these initial conditions. In this interpretation the Euclidean computations are more directly related to the Lorentzian ones. In this case the closed string radiation that comes out of a decaying brane is basically identical to the state at $t = 0$ of the Euclidean path integral with some D-instantons added at a critical Euclidean distance from the $t = 0$ plane.

In section 2 we review Sen’s description of the boundary state and compute the closed string radiation. In section 3 we explain that the closed string states can be viewed as being produced by D-instantons that are sitting at a critical distance in the Euclidean direction. Section three has some overlap with the discussion in [25]. In section 4 we describe some attempts at computing the backreaction. We have included a few appendices. In appendix A we give a second quantized description of various formulae given in the main text. In appendix B we describe an effective field theory model for the closed string creation following the discussion of [5]. In appendix C, as a warmup for the backreaction question, we discuss a simple example of the problem of finding the deformation of a boundary state due to a closed string deformation of the background.

2. Closed string emission from a rolling tachyon

2.1 Review of Sen's computation

The decay of a Bosonic D-brane was described in [3] as a boundary conformal field theory which is the usual non-decaying D-brane plus a boundary interaction of the form

$$S_{\text{bdy}} = \int_{\sigma=0} d\tau \tilde{\lambda} \cosh X^0(\tau), \tag{2.1}$$

where we have set the boundary of the worldsheet at $\sigma = 0$ and we have set $\alpha' = 1$. This tachyon profile is interpreted as a configuration where the tachyon comes up from the minimum associated with the closed string vacuum, gets closest to zero at $t = 0$ and then rolls back. Other authors [8, 10, 11] have also discussed a seemingly more realistic process where we have a boundary interaction of the form

$$S_{\text{bdy}}^{\text{half}} = \int_{\sigma=0} d\tau \lambda e^{X^0(\tau)}, \tag{2.2}$$

that describes the unstable brane at early times which then decays. We will call (2.1) the full-S-brane and (2.2) the half-S-brane.

The boundary state for a Dp -brane with boundary interaction (2.1), (2.2) takes the form

$$|B\rangle = \mathcal{N}_p |B\rangle_{X^0} \otimes |B\rangle_{\tilde{X}} \otimes |B\rangle_{bc}, \tag{2.3}$$

where the normalization constant

$$\mathcal{N}_p = \pi^{\frac{11}{2}} (2\pi)^{6-p}, \tag{2.4}$$

is the same as that for a non-decaying Dp -brane (corresponding to $\lambda = 0$). $|B\rangle_{\tilde{X}}$ and $|B\rangle_{bc}$ are the usual boundary states for the spatial and ghost part of a flat Dp -brane. $|B\rangle_{X^0}$, which describes the dynamics of the rolling tachyon, has the form

$$|B\rangle_{X^0} = \rho(X^0) |0\rangle + \sigma(X^0) \alpha_{-1}^0 \tilde{\alpha}_{-1}^0 |0\rangle + \dots \tag{2.5}$$

where \dots denotes higher oscillation modes. The functions ρ, σ are given by [3, 10]

$$\begin{aligned} \rho_{\text{full}}(t) &= \frac{1}{1 + \widehat{\lambda} e^t} + \frac{1}{1 + \widehat{\lambda} e^{-t}} - 1, \\ \widehat{\lambda} &= \sin \pi \lambda \\ \sigma_{\text{full}}(t) &= \cos 2\pi \lambda + 1 - \rho_{\text{full}}(t) \end{aligned} \tag{2.6}$$

$$\begin{aligned} \rho_{\text{half}}(t) &= \frac{1}{1 + \widehat{\lambda} e^t}, \\ \widehat{\lambda} &= 2\pi \lambda \\ \sigma_{\text{half}}(t) &= 2 - \rho_{\text{half}}(t) \end{aligned} \tag{2.7}$$

for the full-brane and half-brane respectively.

Some essential aspects of the boundary state (2.3) can be summarized as follows [3, 4, 10, 11]:

- i) If $\lambda < 0$ the system becomes singular at a finite value of t , as can be seen from (2.6)–(2.7). This is interpreted as due to the fact that the potential is unbounded from below in Bosonic string theory and the tachyon can run off on the “wrong side”, i.e. away from the closed string vacuum. This does not occur in the more realistic case of the superstring. In this paper we restrict to $\lambda > 0$.
- ii) For the half-brane λ is not a parameter, it can be set to 1 by a time translation. The full-brane system is periodic in λ and it can be restricted to lie between $0 \leq \lambda \leq \frac{1}{2}$. The half-brane can be obtained from the full-brane as a limit of $\lambda \rightarrow 0$ along with a time translation.
- iii) For the full-brane, at $\lambda = \frac{1}{2}$, the boundary state vanishes identically for real values of time. This can be interpreted as saying that the system sits at the closed string vacuum. For λ lying between 0 and $\frac{1}{2}$, the larger the value, the further away the turning point is from the top of the tachyon potential.
- iv) During the rolling process the energy is conserved and the stress tensor of the system can be written as

$$T_{00} = \frac{1}{2}T_p(\rho(t) + \sigma(t)), \quad T_{ij} = -T_p\rho(t)\delta_{ij}, \quad (2.8)$$

where i, j denote spatial directions of the worldvolume and all other components of the stress tensor vanish. In (2.8) T_p is the tension of a Dp-brane and ρ, σ are given by (2.6)–(2.7). It follows from (2.6)–(2.7) that as $t \rightarrow \infty$, $T_{ij} \rightarrow 0$, i.e. the system becomes pressureless.

- v) The formula for $\rho(t)$ can be obtained¹ from an effective field theory model [26, 27] for the tachyon with the action $S = -T_p \int V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T)}$ with the potential

$$V(T) = \frac{1}{\cosh \frac{T}{2}}, \quad (2.9)$$

as we detail in appendix B, see also [22, 23, 28, 29].

2.2 Closed string radiation

A D-brane acts as a source for closed string modes. With a rolling tachyon the D-brane becomes a *time-dependent* source. Thus generically there will be closed string creation from the rolling process. If we ignore the interactions between closed strings, the question then reduces to the familiar problem of free particle production by a time-dependent source. The created state is a coherent state. Schematically, if a^\dagger is the creation operator for a *physical* closed string mode, then the closed string state at late times is proportional to $e^{\alpha a^\dagger} |0\rangle$ where α is essentially the one-point amplitude of the closed string mode on the disk. More precisely,

$$\sqrt{2E}\alpha = \mathcal{A} = \mathcal{N}_p \langle V \rangle_{\text{disk}}, \quad (2.10)$$

¹Note the relation between λ and T_{00} in this effective field theory is different from that in string theory.

where E is the spacetime energy of the state and V is the corresponding on-shell vertex operator. \mathcal{N}_p is the normalization factor (2.4). Notice that this is a process which is of order g^0 .

It is possible to show that for any physical on-shell closed string state with *nonzero* energy we can choose a gauge where there are no timelike oscillators, see [30]. In such a gauge the full vertex operator for an on-shell closed string state can be chosen to be of the form

$$V = e^{iEX^0} V_{sp} \tag{2.11}$$

where V_{sp} is made out of the 25 spatial fields. This is equivalent to the statement that we can choose a gauge where we can put all the α^0 oscillators to zero. It should be noted that the vertex operator V_{sp} is constrained to be a Virasoro primary state of arbitrary conformal weight Δ . The L_0 constraint gives us the value of the energy E in terms of Δ , $E = 2\sqrt{(\Delta - 1)}$. The gauge (2.11) is very convenient since the computation of the one point function factorizes as

$$\langle V \rangle_{\text{disk}} = \langle e^{iEX^0} \rangle_{X^0} \langle V_{sp} \rangle_{sp} \tag{2.12}$$

into a product of the one point function for the time part and a one point function for the rest.

The computation of the second factor in (2.12) is straightforward and it is given by the inner product of the spatial part of the boundary state, which is the spatial part of a standard Dp -brane, with the closed string state under consideration. The spatial part of the boundary state has the form, up to an overall constant,

$$\int d^{25-p}k \sum_{\psi \in H_L} e^{i\varphi(\psi)} |\psi\rangle_L |\psi\rangle_R |k\rangle. \tag{2.13}$$

Note that the boundary state is such that we have the same oscillator state for the right and left movers, up to a phase that is irrelevant for our computations. We also have a sum over all possible oscillator states we can make with the left movers. So the counting of states is the same as the counting for an open string. Finally, all states appearing in (2.13) are unit normalized.

We can separate the closed string Hilbert space into states that are left right identical, as in (2.13) and the orthogonal complement. So only states in the first subspace will be emitted. If we take $V_{sp} = |\psi^l\rangle_L |\psi^l\rangle_R |k\rangle$ the overlap with (2.13) is just a phase.

Therefore, it follows from (2.12) that up to a phase, (2.10) can be identified with the disk one point function for an operator of the form e^{iEX^0}

$$\mathcal{A} = \mathcal{N}_p I(E) = \mathcal{N}_p \left\langle e^{iEX^0} \right\rangle_{\text{disk}}. \tag{2.14}$$

To evaluate (2.14) it is convenient to separate the zero mode integration with the decomposition $X^0 = t + \hat{X}^0$ and we have

$$I(E) = \left\langle e^{iEX^0} \right\rangle_{\text{disk}} = \int_{\mathcal{C}} dt e^{iEt} \left\langle e^{iE\hat{X}^0} \right\rangle_{\text{disk}}. \tag{2.15}$$

Note that in performing the zero mode integration one has to specify a contour \mathcal{C} . The computation of $\langle e^{iE\hat{X}^0} \rangle_{\text{disk}}$ essentially reduces (almost by definition) to that of computing the first term in (2.5). Thus we find that

$$I(E) = i \int_{\mathcal{C}} dt \rho(t) e^{iEt} \tag{2.16}$$

where $\rho(t)$ is given by (2.6) or (2.7). In order to determine the contour \mathcal{C} we need some more physical information. The most obvious choice of contour is to take it to run along the real axis $-\infty < t < +\infty$. We call this the *real* contour, $\mathcal{C}_{\text{real}}$. This contour corresponds to interpreting the full-brane as a configuration where the tachyon rises up to a value closest to zero and then decays back again. Having the tachyon rise in this way in the full theory involves some fine-tuning which we do not worry about here. For the half-brane it is necessary to rotate $t \rightarrow t(1-i\epsilon)$ in order to make the integral convergent. This corresponds to the standard choice of vacuum for closed strings, i.e. there is no incoming radiation from $t = -\infty$.

Since the full-brane interaction (2.1) is time symmetric around $t = 0$, there is another interesting contour which we will call the Hartle-Hawking contour, \mathcal{C}_{HH} . This contour runs from $t = i\infty$ along the imaginary axis to $t = 0$ and then along the real axis to $t = +\infty$. We can imagine the contour along the imaginary axis as preparing the state at $t = 0$. If we did this in a field theory in the bulk the path integral up to $t = 0$ along the imaginary axis would produce a state at $t = 0$. This state would be the vacuum if there were no brane. When there is a brane the state is not the vacuum but some other state produced by the brane. We then evolve this state for $t > 0$ along the real axis. Note that this prescription also creates a state for the open string modes on the brane. This is the state that we naturally get if we blindly analytically continue the Euclidean expressions. We will elaborate more about the physical interpretation of the Hartle-Hawking contour in the next section.

The integral (2.16) for the different contours gives

$$\begin{aligned} i \int_{\mathcal{C}_{\text{real}}} dt \rho_{\text{full}}(t) e^{iEt} &= (e^{-iE \log \hat{\lambda}} - e^{iE \log \hat{\lambda}}) \frac{\pi}{\sinh \pi E}, \\ i \int_{\mathcal{C}_{\text{real}}} dt \rho_{\text{half}}(t) e^{iEt} &= e^{-iE \log \hat{\lambda}} \frac{\pi}{\sinh \pi E}, \\ i \int_{\mathcal{C}_{\text{HH}}} dt \rho_{\text{full}}(t) e^{iEt} &= e^{-iE \log \hat{\lambda}} \frac{\pi}{\sinh \pi E}. \end{aligned} \tag{2.17}$$

The two exponentials in the first line of (2.17) can be interpreted as the radiation emitted in the rising part of the full-brane and the decaying part of the full-brane. Note that the $\hat{\lambda}$ dependence for the half brane is precisely what we expect on the basis of time translation invariance.

Surprisingly, the result for the full-brane using the Hartle-Hawking contour gives the same result as the half-brane. From (2.10) and (2.14) this implies that, up to a phase due to time translation, the final closed string state produced by the half-brane is the same as that produced by the full-brane using the Hartle-Hawking contour. Since $\hat{\lambda}$ only appears

in the phase, the closed string states with different values of $\hat{\lambda}$ only differ by an overall time translation. For the half-brane this is a trivial consequence of the fact that changing λ is the same as performing a time translation, but for the full-brane with $\mathcal{C}_{\mathcal{HH}}$ this is surprising. Note that the Hartle-Hawking prescription leads to a time symmetric state. So we have radiation coming in from $t = -\infty$ and being absorbed by the brane. For $\lambda = 1/2$ we have no D-brane and this radiation that comes from $t = -\infty$ just reflects from the origin and goes out. In contrast, for the full-brane with the real contour, the absolute value of the amplitude is λ dependent and vanishes at $\lambda = \frac{1}{2}$. For $\lambda \rightarrow 0$ we see that the closed strings produced by the rising phase and decaying phase is moved all the way to $t = \mp\infty$.

Note also that the contour that is natural in the Euclidean theory, which runs along the imaginary axis, would be badly divergent for real E . Nevertheless (2.17) are analytic in E and when $E = in$ they have a pole. We see that the residue of the pole for the Hartle-Hawking contour or the half-brane has the values we get in the Euclidean computation. The fact that there is a pole is related to the fact that the Euclidean computation for $E = in$ has a divergent volume factor, the volume of Euclidean time. Note that that the residue of the pole for the real contour is *not* the same as the result for the Euclidean computation. As we explained above the Euclidean computation is more directly related to the Hartle-Hawking contour.

In conclusion the emission amplitude for this closed string state will be given by the integrals (2.17) with E equal to the spacetime energy of the state.

Now we compute the total average number and the total energy of particles emitted. Since the final state is a coherent state, from (2.10), (2.14) we have

$$\frac{\bar{N}}{V_p} = \sum_s \frac{1}{2E_s} |\mathcal{A}_s|^2 = \mathcal{N}_p^2 \sum_s \frac{1}{2E_s} |I(E_s)|^2, \tag{2.18}$$

$$\frac{\bar{E}}{V_p} = \sum_s \frac{1}{2} |\mathcal{A}_s|^2 = \mathcal{N}_p^2 \sum_s \frac{1}{2} |I(E_s)|^2, \tag{2.19}$$

where the sum runs over a basis of physical left-right identical closed string states and V_p is the spatial volume of the Dp -brane. Note that the sum over s includes both the sum over level n and an integral over the momenta of the spatial directions transverse to the brane.

It is interesting to estimate the behaviour of (2.18) and (2.19) for large n to check if the result is finite or not. We see that

$$|I(E)|^2 \sim e^{-2\pi E}, \quad D(n) \sim \frac{1}{\sqrt{2}} n^{-\frac{27}{4}} e^{4\pi\sqrt{n}}, \tag{2.20}$$

where $D(n)$ is the number of primary closed string oscillator states that are left-right identical at level n . This number goes as the number of states in the open string Hilbert space, see [31]. Note that the boundary state in the spatial directions (2.13) contains 25 oscillators, while the degeneracy in (2.20) grows as that of 24 oscillators. This is due to the fact that the condition that the state is a Virasoro primary eliminates precisely one oscillator worth of states. This is due to the fact that the Virasoro characters for $c > 1$

and $\Delta > 0$ have no null states so that their counting is precisely the same as the counting of a single oscillator.

The energy is related to the level n by

$$E = \sqrt{\vec{k}_\perp^2 + 4n} \sim 2\sqrt{n} + \frac{\vec{k}_\perp^2}{4\sqrt{n}} + \dots \tag{2.21}$$

where we have expanded the result for large levels. When we insert this in (2.18) and (2.19) we find that the exponential terms are exactly cancelled. In addition the integral over \vec{k}_\perp produces factors of $n^{1/4}$ per additional transverse dimension. The result takes the form

$$\frac{\bar{N}}{V_p} \sim \sum_n n^{-\frac{p}{4}-1}, \quad \frac{\bar{E}}{V_p} \sim \sum_n n^{-\frac{p}{4}-\frac{1}{2}}. \tag{2.22}$$

Note that the emitted energy is finite for $p > 2$ and infinite for $p \leq 2$. In the cases that the expression for the energy is finite we find that the total energy emitted is of order g^0 (in $\alpha' = 1$ units). In the divergent cases the sum is dominated by the large n terms and (2.18), (2.19) can be written as

$$\begin{aligned} \frac{\bar{N}}{V_p} &= (2\pi)^{-p} \int \frac{dn}{2n} (4n)^{-\frac{p}{4}} + \dots \\ \frac{\bar{E}}{V_p} &= (2\pi)^{-p} \int dE E^{-\frac{p}{2}} + \dots, \end{aligned} \tag{2.23}$$

where we restored the correct numerical factors.

Of particular interest is the $p = 0$ case. In this case the number of emitted particles diverges logarithmically and the emitted energy diverges linearly. If we cutoff the energy at $E \sim 1/g$ we see that we get an energy of the same order of magnitude as the initial mass of the brane. Of course the energy emitted cannot be greater than the mass of the initial brane, so higher order corrections must cut it off so that the total emitted energy is finite. What we find suggestive is that energies of order $E \sim 1/g$ are precisely the energies where we expect the tree level computation to become invalid, since at this energy the gravitational force between the emitted state and the original D-brane become of order 1.

2.3 Remarks on the fate of branes

The above computation strongly suggests that a D0-brane decays completely into closed strings. Note that according to the formulas (2.23) most of the energy is emitted into very massive string states. These massive string states are roughly the same as the ones that appear in the boundary state, so they represent closed strings highly localized at $r = 0$ in the transverse directions. Since their momentum is of order $n^{1/4}$ and mass of order $n^{1/2}$, they move very slowly at the speed of $n^{-1/4}$ away from the origin. Thus the time scale for having some matter localized within string scale around the origin is of order $g^{-1/2}$.

For a D p -brane ($p > 2$) with an noncompact worldvolume (or in a very large compact space) equations in (2.23) appear to suggest that the amount of energy going into closed string modes is rather small and one might conclude that the tachyon matter is stable at

least in perturbation theory.² In order to understand this more precisely it is useful to consider the case where we compactify the Dp brane on a very large volume. In that case, we could also consider the decay into winding modes. If we also include these winding modes, then the problem is similar to considering a D0 brane decaying on a very small compact space. For a D0 brane the mean value of the transverse momentum for a string at level n is of order $k_{\perp} \sim n^{1/4}$. For a D0 brane we expect that \sqrt{n} is of the order of the initial energy of the D0 brane, $1/g_0 \sim \sqrt{n}$. Thus, most of the energy will come out in states which have a momentum of order $k_{\perp} \sim \frac{1}{\sqrt{g_0}}$. In order for this computation to be well defined we need that this momentum is much bigger than the inverse size of the space. If we started originally with a Dp brane on a torus with all circles of radius R_p , then we find that the associated D0 brane lives on a space of size $R_0 = 1/R_p$ and the string coupling is $g_0 = g_p/(R_p)^p$. Thus, we see that for small fixed coupling g_p and very large sizes R_p , then we always find that, for $p \geq 2$, the mean momentum (in the D0) brane picture is $k_{\perp} \sim \frac{(R_p)^{\frac{p}{2}}}{\sqrt{g_p}} \gg \frac{1}{R_0} = R_p$. Note that g_0 is also very small in the large R_p limit. When we go to the original Dp brane picture this is saying two things. First, it is saying that the total energy is still divergent if we include winding modes. Second, that most of the energy is coming out in winding modes. These are strings that are wound along the large directions of the torus. These strings still give the dominant contribution to the energy even in the infinite volume limit, where these strings are infinitely long.

We would like to emphasize that the homogeneous decay as in (2.1) and (2.2) is hardly physical for $p \geq 1$. In these cases, long wavelength tachyonic modes with $k^2 < 1$ are also unstable and will grow in time [6, 10]. This means that the D-brane decay will be rather inhomogeneous. The energy density of the D-brane will tend to concentrate around the minima of the potential and the original Dp-brane may be thought of decomposing into a collection of small patches around various minima, see [6, 10].

2.4 Superstring case

The boundary state for an unstable D-brane in superstring theory is rather similar to the bosonic one and was discussed in [4, 10]. The basic results are also rather similar. Indeed the profiles ρ_{full} and ρ_{half} are of the same form. The on-shell tachyon has the form $T = \lambda e^{\pm X^0/\sqrt{2}}$ and we get an extra square root of two in the exponents for the profiles (2.6) (2.7). Therefore the Fourier transforms have a weaker exponential fall off as a function of the energy. However the exponential growth in the density of states for the superstring is also slightly slower and again the two cancel so that \bar{N} and \bar{E} have exactly the same form as (2.22). Hence the emitted energy also diverges linearly with energy for a decaying unstable D0-brane. Note that in the bosonic string the inhomogeneous decay is potentially ill-defined since tachyon can also be negative. In particular one might worry that small fluctuations near the top of the tachyon potential could send parts of the Dp-brane towards the vacuum with unbounded energy, rather than the closed string vacuum. For superstrings this is not a problem since the potential is bounded from below.

²This paragraph was added in Feb. 2007 thanks to comments by J. Polchinski and it improves our original discussion.

2.5 One loop diagram

In the previous subsection we computed the closed string radiation in a rather direct way by computing the one point functions that can be computed from a variety of methods [3, 10, 11].

The D-brane state acts as a classical source for closed strings. So the one loop Feynman amplitude would give us information about the radiation that comes out. Equations (2.18) and (2.19) can be precisely understood this way and we give a derivation based on the second quantized theory in appendix A. The formal expression for the one loop amplitude in the closed string channel is

$$W = \frac{1}{2} \left\langle B \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0 - i\epsilon} \right| B \right\rangle. \quad (2.24)$$

One can show that the imaginary part of the above one loop effective action is related to the total number of particles radiated, i.e.

$$\bar{N} = 2\text{Im}W = \text{Im} \left\langle B \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0 - i\epsilon} \right| B \right\rangle. \quad (2.25)$$

Similarly the energy emitted can be written as

$$\bar{E} = \frac{\delta}{\delta a} \left\langle B \left| \left(\frac{b_0^+ c_0^-}{L_0 + \bar{L}_0} \right)_{\text{ret}} \right| B(a) \right\rangle \Big|_{a=0}, \quad (2.26)$$

where $|B(a)\rangle$ is obtained from $|B\rangle$ by taking $t \rightarrow t + a$, i.e. they are separated by an interval a in time direction. The subscript “ret” indicates that one needs to use the retarded propagator. One can check that, level by level, (2.25) and (2.26) agree with (2.18) and (2.19).

For a static D-brane, (2.24) is related by modular invariance to the one-loop partition sum of the open string theory. In our case, such a relation is not clear since we have a Lorentzian cylinder. However, one can show that the imaginary part (2.25) does have an interesting open string interpretation. More explicitly, one finds that after summing over all levels, for the half-brane, (2.25) can be written in the open string form

$$\frac{\bar{N}}{V_p} = \sum_{n=1}^{\infty} n \int_0^{\infty} \frac{ds}{s} \frac{1}{(4\pi\alpha's)^{\frac{p}{2}}} e^{-n^2 s} \frac{1}{\eta^{24} \left(\frac{is}{2\pi} \right)}. \quad (2.27)$$

The above equation can also be rewritten as the following

$$\frac{\bar{N}}{V_p} = \left\langle B_- \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0} \right| B_+ \right\rangle, \quad (2.28)$$

where $|B_{\pm}\rangle$ are defined by

$$|B_+\rangle = \sum_{n=0}^{\infty} |B_p[x = (2n+1)\pi]\rangle, \quad |B_-\rangle = \sum_{n=0}^{\infty} |B_p[x = -(2n+1)\pi]\rangle, \quad (2.29)$$

with $|B_p[x]\rangle$ the boundary state of a *Euclidean* D-instanton with p spatial dimensions sitting at position x in the Euclidean time direction. We see that (2.28) includes only open strings which stretch across the $t = 0$ line. This is different than the naive Euclidean computation for an array of D-instantons (corresponding to $\lambda = 1/2$ for the full-brane), which would include open strings stretching between any two D-instantons. Note that we get the result (2.28) for the half-brane or the full-brane with Hartle-Hawking contour independently of the value of λ . Note also that the overlaps of boundary states include both orientations of stretched open strings. The corresponding formula for the full-brane with the real contour is somewhat more complicated, and can be obtained by replacing $e^{-n^2 s}$ in (2.27) by

$$2e^{-n^2 s} - \exp \left[\left(\frac{\log \hat{\lambda}}{\pi} - in \right)^2 s \right] - \exp \left[\left(\frac{\log \hat{\lambda}}{\pi} + in \right)^2 s \right]. \quad (2.30)$$

It is not clear whether the above equation has an open string interpretation. The expression (2.30) appears to come from D-instantons sitting at complex positions in the complex t plane.

Another way to derive (2.28) is as follows. Note that for the half-brane we can rewrite (2.14) and (2.17) as

$$\begin{aligned} \mathcal{A} &= N_p I(E) = N_p \left\langle e^{iEX^0} \right\rangle_{\oint \lambda e^{X^0}} \\ &= 2\pi N_p e^{-iE \log \hat{\lambda}} \sum_{m=0}^{\infty} e^{-\pi(2m+1)E} \\ &= e^{-iE \log \hat{\lambda}} \sum_{m=0}^{\infty} N_{p-1} \left\langle e^{iEX^0} \right\rangle_{X^0=i(2m+1)\pi} \end{aligned} \quad (2.31)$$

where in the last line above $\langle \rangle$ denotes one point function for a D-instanton sitting at a point on the imaginary time axis. Thus when we sum over all intermediate closed string modes in (2.18) we find (2.28). The appearance of D-instantons can be understood as follows. In obtaining $\rho(t)$ (2.7) in (2.16) we treat the worldsheet boundary interaction (2.2) in terms of perturbative expansion

$$\sum_n \frac{1}{n!} \left(\oint d\tau \lambda e^{X^0} \right)^n$$

However, the perturbation series only have a finite radius of convergence and this is reflected in the singularities of $\rho(t)$ on the complex plane. For example, $\rho(t)$ in (2.7) has poles at $t = i(2m+1)\pi + \log \hat{\lambda}$. The contributions of these poles to the integrals (2.17) are precisely given by the D-instantons discussed in (2.31). When there are poles on real axis, as is the case for $\hat{\lambda} < 0$ in (2.6), (2.7) and some inhomogeneous decay examples [6, 10], (2.16) is not unambiguously defined.

Equation (2.27) gives a very clean picture of the divergence in \bar{N} for $p = 0$ in terms of the open string picture. This is an *IR divergence* (arising at large s) that comes from the open string stretching between the two closest D-instantons sitting at $x = \pm\pi$. The

distance between the two D-instantons is such that this open string is precisely massless. This gives rise to the divergence.

The expression for the energy can be obtained by considering

$$\frac{\bar{E}}{V_p} = \frac{\partial}{\partial a} \left\langle B_- \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0} \right| B_+(a) \right\rangle \Big|_{a=0}, \quad (2.32)$$

where $|B_+(a)\rangle$ corresponds to the same array we had in (2.29) but shifted by a in the Euclidean time direction. In other words, the open strings that had masses squared proportional to $(2\pi n)^2 - 1$ have masses proportional to $(2\pi n + a)^2 - 1$ after we shift $|B_+\rangle$ by a .

If $0 < p \leq 2$ we get a divergence in the energy which is related to the fact that a massless field (the massless open string mode stretching between the two D-instantons closest to $t = 0$) does not have a well defined vacuum in two or less Euclidean dimensions.

Note that if we treated the closed string theory as an ordinary field theory we would have to sum over real Lorentzian momenta in the time direction. In other words the one loop diagram becomes

$$\mathcal{Z}_1 = \mathcal{N}_p^2 \sum_n \int \frac{dk}{2\pi} \frac{1}{-k^2 + E_n^2 - i\epsilon} I(k)I(-k), \quad (2.33)$$

where the sum is over all left-right identical closed string states with energies E_n . As above the sum over n includes an integral over the transverse directions. It is easy to see that, level by level, the imaginary part of (2.33) indeed agrees with the expression for \bar{N} (2.18). The normalization factor can be computed by noticing that for $\lambda \rightarrow 0$ we should recover the usual expression for Bosonic D-branes [32].

Note that the expression (2.33) does not include possible $\delta(k)$ contributions which according to Sen, [3], are present in the boundary state. One example is the contribution that gives rise to the conserved energy. The Euclidean boundary state also contains contributions proportional to $\delta(k - in)$, it is not clear if these should be included or not.

In [8, 11], open string creation was discussed. One would expect this to be related by modular invariance to the computations done in this paper. We could not make this relation explicit. The amount of energy that was lost into open string modes, according to [8, 11], grows exponentially with time, with a coefficient that is finite for $p < 25$ and infinite for $p = 25$. Computations in the open string description seem harder because we do not expect to have asymptotic physical open string states once the tachyon condenses. On the other hand, closed string states are still well defined in the future.

Note that for the full brane with the Hartle-Hawking contour the state that is emitted depends on λ only through a simple phase. So the only difference in the state emitted is an overall time translation. Since the state is time reflection symmetric the incoming closed strings are also translated by a similar amount into negative times. This is schematically represented in 1. If we assume that the D0-brane decays completely into these closed string modes, then we get an intriguing picture where finely tuned incoming closed strings produce an unstable D0 which after a while decays again into closed strings.

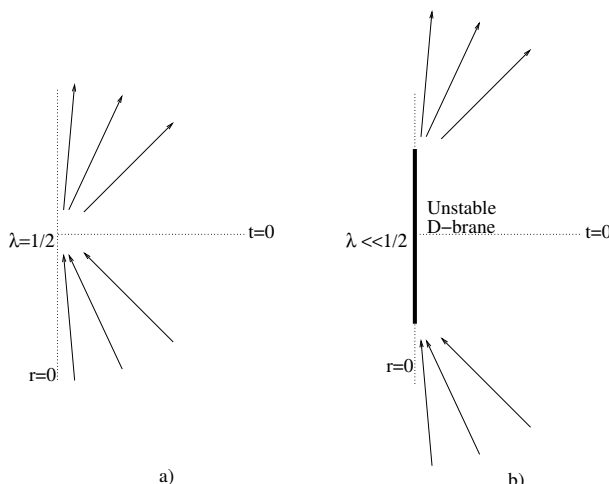


Figure 1: Lorentzian picture for the Hartle-Hawking state for the full brane. The state is time reflection symmetric. We have incoming closed string radiation from the past and outgoing in the future. It is indicated by arrows. In a) we show the state for $\lambda = 1/2$ which contains only incoming and outgoing radiation. In b) we show the state for $\lambda < 1/2$, the incoming radiation is absorbed by the brane and then reemitted.

3. D-instantons and the closed string final state

There is a very beautiful and simple description of the closed string state that is produced by the full-brane with the Hartle-Hawking contour or the half-brane.³ As we explained above the state that is produced is a coherent state in the space of closed string fields. Let us think for the moment about the bulk closed string field theory as an ordinary field theory. A method for producing coherent states is to perform the path integral in Euclidean space with a source and cut this path integral at $t = 0$. The result of the path integral is a function of the boundary values of the fields at $t = 0$. We can think of this function as a wavefunction for the fields. For example, the wavefunction of the vacuum is obtained by doing the path integral over negative imaginary Euclidean time. If we insert operators or sources at Euclidean times $t_E < 0$ this same path integral will produce a different state.

For simplicity let us start by discussing the state that we get if we insert a D-instanton at a distance d from $t = 0$ (see figure 2). This will act as a point source for many closed string fields which at $t = 0$ will have an amplitude of the form e^{-dE} where E is the energy of the particular closed string mode. The norm of the state involves squaring the amplitude and summing over all states. The terms in this sum go as $e^{-2dE} e^{2\pi E}$ for large E . So see that if $d < \pi$ the state that we produce in this way is not normalizable. Therefore the approximation in which we derived it is not valid and we do not trust the resulting state. On the other hand if $d > \pi$, the state that we create in this fashion is a perfectly nice and normalizable finite energy state in the field theory of closed string modes and has an

³This discussion has some overlap with the discussion in [25], where these issues are discussed in more detail.

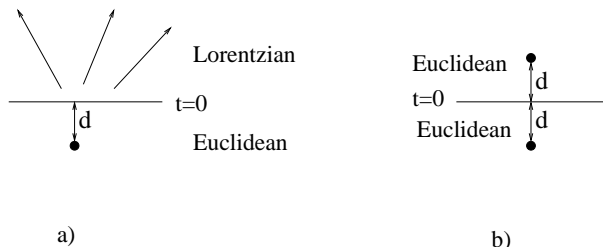


Figure 2: Construction of a closed string state via the insertion of D-instantons in Euclidean space. In a) we pictorially represent doing the path integral over half of the Euclidean time line, $t_E < 0$, with an instanton inserted and in b) we represent the computation of the norm of the state. This norm becomes ill defined if the D instanton distance to $t = 0$ is $d < \pi$.

energy of order one (i.e. g^0). We can understand this change in behaviour as follows. We can compute the norm of the state produced in the above way by considering the path integral over full Euclidean time were we put sources at $t > 0$ in a time reflection symmetric fashion. The divergence in the norm is related to the fact that the open string going between the D-instanton at $t = -d$ and the one at $t = +d$ becomes tachyonic precisely at $d = \pi$. So this divergence is rather similar to the Hagedorn divergence. Note, however, that the state that is produced in this fashion is not thermal, it is a pure state.

Now, instead of a single D-instanton we can consider an array of D-instantons sitting at $t = -i\pi(1 + 2m)$, $m = 0, 1, 2, \dots$. The large energy behaviour of the amplitudes is dominated by the D-instanton that is closest to $t = 0$. It is interesting that the states arising during tachyon decay correspond to D-branes at $d = \pi$ in Euclidean space. When we compute the norm of the state we have the array of D instantons at $t_E = i\pi(1 + 2m)$, $m \in \mathbb{Z}$. This is precisely the Euclidean state for $\lambda = 1/2$, for which there is no brane in Lorentzian signature [3]. However, when we computed the closed string radiation in (2.17) we found some radiation coming out if we used the Hartle-Hawking contour. But this radiation is the same radiation that is coming in from $t = -\infty$ in a time symmetric fashion, see 1. The state at $t = 0$ contains no brane because the D-instantons are localized in Euclidean time away from $t = 0$. Note that this description is completely explicit. For example we can consider the dilaton field. Its expectation value in Euclidean space is given by the expression (see also [12])

$$\phi(r, t_E) = \sum_{m=-\infty}^{\infty} \frac{1}{[r^2 + (t_E - \pi(1 + 2m))^2]^{\frac{24-p}{2}}} . \quad (3.1)$$

which in Lorentzian signature gives

$$\phi(r, t_L) = \sum_{m=-\infty}^{\infty} \frac{1}{[r^2 + (it_L - \pi(1 + 2m))^2]^{\frac{24-p}{2}}} . \quad (3.2)$$

If we compute the positive frequency components of (3.2) we find again an expression like (2.17) with $E = |k|$ where k is the momentum in the directions transverse to the D-instanton used to produce (3.1). This D-instanton has p spatial dimensions. As

expected (3.2) obeys the massless, sourceless field equation in the bulk. It represents a dilaton wave coming in to $r = 0$ and then going back to $r \rightarrow \infty$ (see 1).

Our previous formulas describe completely the coherent state that is produced by this process. Just to clarify the situation a bit more let us point out that the closed string state that is produced by a D-instanton sitting at position d in Euclidean space is given by

$$|C\rangle = P_{\text{phys}} e^{-d\hat{E}} |B\rangle \tag{3.3}$$

where $|B\rangle$ is the boundary state for a D-instanton at the origin. \hat{E} is the operator that measures the energy of the state. And finally P_{phys} is a projector on to physical states. The state $|C\rangle$ is a state in the closed string Hilbert space as long as $d > \pi$. The actual state in the full multiparticle closed string Hilbert space is the coherent state associated to the state (3.3). In other words, if we call $\alpha\alpha^\dagger$ the operator creating a single string in the state (3.3), then the full state is $e^{\alpha\alpha^\dagger}|0\rangle$.

For the superstring, at $\lambda = 1/2$, the corresponding Euclidean configuration involves an array of alternating D-instantons and anti-D-instantons. This is due to the fact that the RR fields get an i when we continue to Euclidean signature so they need to get a minus sign when we perform a time reflection in Euclidean signature in order to be real in Lorentzian signature. The alternating charges ensure that.

Note that these instanton produced states are in the spirit of the S-branes discussed in [2] but differ from the D-s-branes localized in lorentzian time that were discussed in [2] by the crucial fact that they are displaced by a critical distance in Euclidean space. This displacement makes sure that the state that is produced does not have a badly divergent norm. If we go back to Lorentzian signature the closed string state that is created this way is time reflection symmetric around the origin. Note also that the state we produce is not $SO(26 - p)$ invariant in Euclidean space, so it does not have $SO(1, 25 - p)$ symmetry in Lorentzian signature.

There is one interesting aspect that makes the string theory computation different from a field theory computation. In a field theory we are free to add any sources we want when we do the Euclidean path integral. In string theory it is not clear that one could do that since one does not have local operators. The D-instantons are objects in the theory that in the classical limit act as heavy fixed sources for the closed string fields. But as we go to higher orders the D-instantons become dynamical and can, for example, move. It would be nice to understand more precisely how to view the setup that we have been describing in a more non-perturbative way. One would have to find a natural context in which one can produce these states. One possibility is to consider the thermal partition function, for temperatures $\beta = 2\pi n$ (we need $n > 2$ in order to be at temperatures lower than the Hagedorn temperature). See [12] for further discussion. This thermal partition function contains contributions from the Euclidean branes with arbitrary λ (in fact we should integrate over λ when we compute the thermal partition function). We could then imagine that we somehow analytically continue these to Lorentzian signature. So that we interpret the process as the spontaneous nucleation of such a brane in the thermal background which then subsequently decays.

In [12] a different interpretation was proposed for the $\lambda = 1/2$ state. This difference is sharper for $p > 2$. In [12] it was proposed that such a state carries an energy density of order $1/g$. Here we are saying that this state carries an energy density of order 1. For $p = 0$ we find a divergent energy and in fact we expect that this divergence is cutoff at an energy of order $1/g$.⁴

4. Attempts of computing the back reaction

For a D0-brane we observed that the one loop diagram diverges.⁵ When such a thing happens in string theory, one is supposed to change the classical string solution in order to cancel the divergence [33]. In the examples discussed in [33] the divergence comes from the IR of the closed string channel and consequently one needs to change the IR properties of the closed string background. In our case the divergence comes from the UV of the closed string channel, or the IR of the open string channel. So we conclude that the divergence should be eliminated by modifying the long time behaviour of the open string classical solution.

We know that Sen's state obeys the classical open string field theory equations. In principle one would like to solve the one-loop corrected open string equations of motion. This seems to be a rather difficult task. Some of the difficulties were described in a related context in [34]. These difficulties come from the fact that we need to evaluate both the disk and the cylinder diagram off shell. However we are only interested in the changes of the open string solution over relatively long time scales. One could object that once one tries to balance classical equations with one loop corrections one needs to include all loops. What makes the situation a little more hopeful in our case is that the emission of very massive closed string modes might be a relatively slow process and could change the boundary state rather slowly.

In any case, we did not succeed in finding the corrected equations and we will just present a summary of our attempts. Hopefully a more satisfactory solution will be found soon.

Our idea was that the parameter λ which appears in the boundary state will no longer be a constant but it would slowly evolve in time.⁶ This would have two effects. One would be to provide an effective cutoff for the energy of the emitted radiation. The second is that the boundary state would slowly approach $\lambda = 1/2$ for the full-brane which corresponds

⁴In fact, the discussion in [12] could also be carried out with a D-instanton array where the instantons are separated by a distance bigger than the critical distance. In that case our description would give, in the Lorentzian theory, a state of energy of order g^0 while a computation in the spirit of [12] would suggest a state with energy proportional to $1/g$. We think that the discrepancy is related to the different translation between the Euclidean and Lorentzian computation.

⁵For higher Dp -branes there are divergences in the expectation values of the energy, or higher powers of the energy. Since these are measurable observables that should give finite answers, we expect corrections for these too.

⁶More precisely we write the boundary interaction on the worldsheet as $S = \int \lambda_+ e^t + \lambda_- e^{-t}$ and assume that λ_{\pm} depend on time.

to having no D-brane, only outgoing radiation. We did not succeed in showing that this is indeed what happens but we will describe the difficulties we ran into.

A basic simple example to keep in mind is the following. Consider a heavy degree of freedom, localized at the origin which couples to a free field that propagates in the bulk. The heavy object is moving along a solution of its classical action. The coupling to the free field would imply that some radiation is produced and this will suck energy away from this degree of freedom. If the action for the total system has the form

$$S = S[q(t)] + \int dt j(q(t))\phi(t, x = 0) + \frac{1}{2} \int dt d^d x (\partial\phi)^2 - m^2\phi^2 \quad (4.1)$$

Then the corrected equations of motion for the particle are

$$\frac{\partial S}{\partial q(t)} + \frac{\partial j}{\partial q(t)} \int dt' G_{\text{ret}}(t, 0; t', 0) j(q(t')) = 0, \quad (4.2)$$

where G is the retarded propagator, it is evaluated at the origin in the transverse dimensions. We would like to find an expression which is equivalent to this in string theory. In principle one could do this in string field theory. In boundary string field theory equations of this type were contemplated in [34] in a similar context.

By taking test functions and integrating them over time against (4.2) we can find a particular subset of equations. A simple one is the energy conservation condition, which arises when we multiply by $\dot{q}(t)$ and integrate (4.2). The resulting equation says that the energy lost by the system is equal to the energy emitted in radiation. So this equation will already be enough to make sure that energy is conserved. In order to get this equation to work in string theory we only need to assume that the classical energy vanishes at late times and that it is proportional to $-\int M_0$ for early times. Then the equation (4.2) implies that

$$M_0 = \bar{E}. \quad (4.3)$$

But this is rather trivial, what we really need to show is that at late times the D-brane state has zero energy.

A simpler problem than the one we have is the following. Suppose you have D-brane in flat space. Now you deform flat space by changing the closed string background. This change is described by adding the operator $\mathcal{O}(z, \bar{z})$ on the string worldsheet. The old boundary state that described the D-brane in flat space is not a good conformal boundary state in this perturbed bulk CFT. In particular, when \mathcal{O} approaches the boundary described by the old boundary condition we will find an operator product expansion of the form

$$\mathcal{O}(z, \bar{z}) \sim \sum_n (z - \bar{z})^{-2+n} V_n \quad (4.4)$$

where V_n are boundary operators of conformal weight n . The term proportional to V_1 will give rise to a logarithmic divergence when we integrate over $z - \bar{z}$. This logarithmic divergence is cancelled by adding to the boundary an interaction of the form

$$\int d\tau V_B(\tau), \quad \text{with } (L_0 - 1)V_B = V_1. \quad (4.5)$$

In appendix C we show how this works for a simple example. In general one would think that we also need to consider the operators V_n with $n \neq 0$. However, the operator V_1 seems the most important one at long distances in spacetime.

Now going back to our problem. When a closed string mode is emitted, the boundary state now moves in the background generated by this closed string mode. So one would attempt to find the correction to the boundary state by performing an analysis like the one above for each closed string state and then summing over all states.

Let us consider first the half-brane. In this case the interesting term in the OPE is

$$e^{inX} \sim \frac{1}{(z - \bar{z})^{n^2-1}} \lambda^n n \left(\frac{J^-}{\lambda} + \frac{1}{3} \lambda J^+ \right). \tag{4.6}$$

when we compute it in the Euclidean theory. $J^3 = i\partial X$ and $J^\pm = e^{\pm iX}$ are the SU(2) generators [35]. It is not obvious precisely how we should analytically continue this to the Lorentzian theory. What is clear is that the analytic continuation of the J^\pm operators involve the boundary tachyon vertex operators $e^{\pm t}$. So after summing over all closed string states we expect to find that V_1 has the form $V_1 = g_s(Ae^t + Be^{-t})$ where A and B are some numerical coefficients. The factor of g_s comes from the fact that the vertex operator for the emitted state comes with a factor of g_s .

Then the operator V_B will have a form proportional to $g_s(Ate^t - Bte^{-t})$, we can interpret this as giving us the time variation for the coefficient λ . In other words, we find $\dot{\lambda} \sim g_s$. Unfortunately we could not find a more precise equation for the effective evolution of $\hat{\lambda}$.

Note that if we have a profile such as (2.7) with a $\hat{\lambda}(t)$ which is a slowly varying function of t , this will have the effect to shift the pole position from $\text{Im}t_p = \pi$ to

$$\text{Im}(t_p) = \pi \left(1 - \frac{\dot{\hat{\lambda}}(0)}{\hat{\lambda}(0)} \right) + \dots, \tag{4.7}$$

where we assumed that derivatives of λ are small and we expanded λ to first order in time. So we see that if $\dot{\lambda} < 0$. Then the position of the pole shifts in such a way that it makes the integral convergent since the exponential suppression factor of the form $e^{-2\text{Im}(t_p)E}$ will be greater than the factor coming from the density of states. Since we expect that $\dot{\hat{\lambda}}$ is of order g_s . This would give the right order of magnitude for the cutoff in energy, i.e. we would get a total emitted energy of order $1/g_s$. In other words, the evolution of $\hat{\lambda}$ is such that the evolution of $\rho(t)$ is slower than the evolution with constant $\hat{\lambda}$. That means that the tachyon vertex operator would be slightly relevant (as opposed to marginal).

When we considered the imaginary part of the one loop diagram in Euclidean space we saw that the divergence came from the open string stretched between the two closest D-branes. This would suggest that in order to make the diagram finite we would need to make this distance slightly bigger than the critical distance. This would suggest that the Euclidean computation would involve the boundary sine Gordon model with a potential slightly relevant.

Finally let us note that if we had a decaying unstable lump in an ordinary weakly coupled field theory it would be enough to solve the classical equations of motion to find

the state it decays into at very late times. To the extent that a D-brane is similar to an ordinary lump solution of string theory one would expect a similar situation. In this case the precise boundary state is the classical solution. So one would expect it to encode the final state that the D-brane decays to. It seems, however, that the situation here is not as straightforward.

5. Conclusion

In this paper we computed the production of closed string modes produced from time-dependent rolling tachyon solutions which represent the homogenous decay (or creation and subsequent decay in the case of the full-brane) of an unstable Dp -brane. For $p > 2$ the total number of particles and total energy radiated away was finite. However in these cases a homogenous decay is very unphysical and Dp -branes will decay inhomogeneously. For $p \leq 2$ we found that the total energy diverges. We interpreted this as an indication that a D0-brane decays completely into closed string modes, in particular mostly massive ones. We outlined a few approaches to the back reaction problem. It would be nice to have a better understanding of it.

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A. Particle production from a source

In this appendix we give a derivation of various formulae in the main section based on a second quantized approach. A closed string field can be written as a linear superposition of basis states:

$$|\Psi\rangle = \sum_s |\Phi_s\rangle \phi_s . \quad (\text{A.1})$$

Each ψ_s , the component of the vector $|\Psi\rangle$ along the basis vector $|\Phi_s\rangle$, corresponds to a *target space field*. In the presence of a boundary state the field equation is given by

$$(Q + \bar{Q})|\Psi\rangle = |B\rangle \quad (\text{A.2})$$

Here B is a vector of ghost number 3 in the state space of closed string CFT which satisfies the conservation law

$$(Q + \bar{Q})|B\rangle = 0 . \quad (\text{A.3})$$

From now on we shall restrict to the cases that the source $|B\rangle$ has the form $|B\rangle = c_0^+ |J\rangle$, with $|J\rangle$ a state of ghost number two. Then after fixing the Siegel gauge $b_0^+ |\Psi\rangle = 0$ we find that

$$(L_0 + \bar{L}_0) |\Psi\rangle = |J\rangle . \quad (\text{A.4})$$

Expanding $|J\rangle$ in components J_s as in (A.1), the equations of motion (A.4) takes the familiar field theory form

$$\frac{1}{2}(\partial^2 - m_s^2)\phi_s = J_s . \quad (\text{A.5})$$

The closed string fields can be quantized using the standard method (e.g. see [36]). We assume that the currents J_s have been switched on only for a finite interval, we may therefore define the “in” and “out” field operators

$$\begin{aligned} \phi_s(x) &= \phi_s^{\text{in}}(x) - 2 \int d^d y G_s^{\text{ret}}(x-y) J_s(y) \\ &= \phi_s^{\text{out}}(x) - 2 \int d^d y G_s^{\text{adv}}(x-y) J_s(y) , \end{aligned} \quad (\text{A.6})$$

where G_s^{ret} and G_s^{adv} are standard retarded and advanced Green functions. In particular if we expand ($E_{s,\vec{p}} = \sqrt{m_s^2 + \vec{p}^2}$)

$$\begin{aligned} \phi_s^{\text{in}}(x) &= \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{1}{\sqrt{2E_{s,\vec{p}}}} \left(a_{s,\vec{p}}^{\text{in}} e^{ip \cdot x} + (a_{s,\vec{p}}^{\text{in}})^\dagger e^{-ip \cdot x} \right) , \\ \phi_s^{\text{out}}(x) &= \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{1}{\sqrt{2E_{s,\vec{p}}}} \left(a_{s,\vec{p}}^{\text{out}} e^{ip \cdot x} + (a_{s,\vec{p}}^{\text{out}})^\dagger e^{-ip \cdot x} \right) , \end{aligned} \quad (\text{A.7})$$

with

$$[a_{s,\vec{p}}^{\text{in}}, (a_{s,\vec{p}'}^{\text{in}})^\dagger] = [a_{s,\vec{p}}^{\text{out}}, (a_{s,\vec{p}'}^{\text{out}})^\dagger] = (2\pi)^{d-1} \delta^{(d-1)}(\vec{p} - \vec{p}') \quad (\text{A.8})$$

then

$$a_{s,\vec{p}}^{\text{out}} = a_{s,\vec{p}}^{\text{in}} - \frac{i}{\sqrt{2E_s}} g_s \tilde{J}_s(-E_{s,\vec{p}}, -\vec{p}) \quad (\text{A.9})$$

where $\tilde{J}_s(p)$ is the Fourier transform of $J_s(x)$ and

$$\tilde{J}_s(E_{s,\vec{p}}, \vec{p}) = \tilde{J}_s(p)|_{p^0=E_{s,\vec{p}}}, \quad E_{s,\vec{p}} = \sqrt{m_s^2 + \vec{p}^2} . \quad (\text{A.10})$$

The S-matrix operator which connects the in- and out-fields and in- and out-states

$$\begin{aligned} \phi_s^{\text{out}}(x) &= \hat{S}^{-1} \phi_s^{\text{in}}(x) \hat{S} \\ |\text{out}\rangle &= \hat{S}^{-1} |\text{in}\rangle, \quad |\text{in}\rangle = \hat{S} |\text{out}\rangle \end{aligned} \quad (\text{A.11})$$

can be written as

$$\begin{aligned} \hat{S} &= T \exp \left[-i \int d^d x \phi^{\text{in}}(x) \cdot J(x) \right] \\ &= : \exp \left[-i \int d^d x \phi^{\text{in}}(x) \cdot \vec{J}(x) \right] : \exp [iW] \end{aligned} \quad (\text{A.12})$$

where

$$W = \frac{1}{2} \left\langle B \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0 - i\epsilon} \right| B \right\rangle \quad (\text{A.13})$$

In the above, T and $::$ denote time and normal ordering respectively, $|\text{in}, 0\rangle$ is the in-vacuum of the second quantized theory and $\vec{\phi} \cdot \vec{J} = \sum_s \phi_s J_s$.

It can be shown from the above that the average number and energy of the particles emitted by the source are given by

$$\begin{aligned} \bar{N} &= \sum_s \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2E_s} \tilde{J}_s(-E_s, -\vec{p}) \tilde{J}_s(E_s, \vec{p}) \\ &= 2\text{Im}W = \text{Im} \left\langle B \left| \frac{b_0^+ c_0^-}{L_0 + \bar{L}_0 - i\epsilon} \right| B \right\rangle \end{aligned} \quad (\text{A.14})$$

and

$$\begin{aligned} \bar{E} &= \frac{1}{2} \sum_s \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \tilde{J}_s(-E_s, -\vec{p}) \tilde{J}_s(E_s, \vec{p}) \\ &= \frac{\delta}{\delta a} \left\langle B \left| \left(\frac{b_0^+ c_0^-}{L_0 + \bar{L}_0} \right)_{\text{ret}} \right| B(a) \right\rangle \Big|_{a=0} \end{aligned} \quad (\text{A.15})$$

where $|B(a)\rangle$ is obtained from $|B\rangle$ by taking $t \rightarrow t + a$, i.e. they are separated by an interval a in time direction. The subscript “ret” indicates that one needs to use the retarded propagator. Note that in (A.14) and (A.15) only on-shell physical modes contribute. This is guaranteed by the conservation law (A.3), which ensures the decoupling of gauge modes.

Note that the probability of going from the in-vacuum to the out-vacuum (i.e. the probability of no emission) is

$$p_0 = |\langle 0, \text{out} | \text{in}, 0 \rangle|^2 = e^{-\bar{N}}. \quad (\text{A.16})$$

When \bar{N} is infinite, the S -matrix does not exist, since every matrix element between in- and out-states vanishes. In a system with an infinite number of degrees of freedom, inequivalent representations of the canonical commutation relation may exist. In certain cases, meaningful physical results can still be extracted, e.g. as in the case of “infrared catastrophe”. In string theory we also have to worry potential divergences from the Hagedorn density of states.

Equation (A.12) means that the final state is a coherent state. From equation (A.6) we have that

$$\langle \text{in}, 0 | \phi_s(x) | \text{in}, 0 \rangle = -2 \int d^d y G_s^{\text{ret}}(x - y) J_s(y). \quad (\text{A.17})$$

Similarly

$$\langle \text{in}, 0 | \phi_s^{(\text{out})}(x) | \text{in}, 0 \rangle = -2 \int d^d y G_s^-(x - y) J_s(y) \quad (\text{A.18})$$

where $G^- = G_{\text{ret}} - G_{\text{adv}}$. Note (A.18) is precisely the classical radiation field. Finally

$$\langle \text{out}, 0 | \phi_s(x) | \text{in}, 0 \rangle = -2 \int d^d y G_s^F(x - y) J_s(y) \quad (\text{A.19})$$

where G^F denotes the Feynmann propagator.

B. Field theory model

In this appendix we consider Sen’s effective field model [27, 26]

$$S = - \int dt V(T) \sqrt{1 - \dot{T}^2} = \int dt L(t) \tag{B.1}$$

for the rolling tachyon. Without loss of generality we will consider the D0-brane theory and take $V(T) = M_0 f(T)$ with M_0 the D0-brane mass. Note that the tachyon field T used here is not the same as that used in elsewhere in this paper but differs by a field redefinition. We will “derive” the tachyon potential $V(T)$ by matching time-dependent solutions of (B.1) to the half-brane solution discussed in the main text.

The equation of motion following from (B.1) is

$$\dot{T} = \sqrt{1 - \frac{V^2}{E^2}}, \tag{B.2}$$

where E is the conserved energy and the Lagrangian evaluated on the solution is given by

$$L = -V \sqrt{1 - \dot{T}^2} = -\frac{V^2}{E}. \tag{B.3}$$

On general grounds we can identify the on-shell value of $-L(t)$ with the partition function on the disk, $M_0 \rho(t)$. For the half-brane, $E = M_0$ and we find

$$\sinh^2 \frac{T}{2} = \hat{\lambda} e^t, \tag{B.4}$$

and

$$f(T) = \frac{1}{\cosh \frac{T}{2}}. \tag{B.5}$$

We note that this potential was also recently used in [22, 23].

With (B.5) we now would like to find the solution to (B.2) for general E

$$\dot{T} = \sqrt{1 - \frac{q^2}{\cosh^2 \frac{T}{2}}}, \quad q = \frac{M_0}{E}. \tag{B.6}$$

When $q > 1$, there is a turning point $\cosh \frac{T}{2} = q$ at which $\dot{T} = 0$. Using time translational symmetry we can set this point to be at $t = 0$. On the other hand when $q < 1$, the tachyon can climb to the top of the potential and we will set this point to be $t = 0$.

Equation (B.6) can be integrated with the above boundary conditions yielding

$$\sinh \frac{T}{2} = \begin{cases} a \cosh \frac{t}{2}, & q > 1 \\ a \sinh \frac{t}{2}, & q < 1 \end{cases} \quad a = \sqrt{|q^2 - 1|}. \tag{B.7}$$

One thus finds

$$-L = \frac{V^2}{E} = M_0 \left(\frac{1}{1 + ce^t} + \frac{1}{1 + ce^{-t}} - 1 \right) = \begin{cases} \frac{M_0^2}{E} \frac{1}{1 + a^2 \cosh^2 \frac{t}{2}} & q > 1 \\ \frac{M_0^2}{E} \frac{1}{1 + a^2 \sinh^2 \frac{t}{2}} & q < 1 \end{cases}, \tag{B.8}$$

with

$$c = \frac{(q-1)^2}{a^2} = \left| \frac{q-1}{q+1} \right| \leq 1 . \tag{B.9}$$

(B.8) is precisely the full-brane profile. Remarkably, despite its non-linear form, this effective action admits the general marginal tachyon deformation, constructed as a linear combination of a half-brane and time reversed half-brane, as a solution to its equation of motion. The $\lambda = \frac{1}{2}$ ($c = 1$) case, where the boundary state vanishes, corresponds in these variables to $T = \infty$, i.e. the tachyon is sitting at the closed string vacuum. Note that while as a function of t (B.8) reproduces the string theory result, the relation (B.9) between c and energy E is different from that in string theory except for the half brane limit.⁷

The potential (B.5) is symmetric under $T \rightarrow -T$ and is bounded below everywhere. Hence it only describes the well behaved half of the potential in Bosonic string theory but it also can be thought of as a model for non-BPS branes in superstring theory. Our analysis before carries over trivially to superstring case by a scaling $T \rightarrow \sqrt{2}T$, $t \rightarrow \sqrt{2}t$, e.g. we have

$$f(T) = \frac{1}{\cosh \frac{T}{\sqrt{2}}} . \tag{B.10}$$

In other words, to apply equations (B.4)–(B.9) to the superstring case, we should set $\alpha' = 2$. When expanding the action (B.1) around $T = 0$ using (B.10), we find $m^2 = -\frac{1}{2}$, precisely that of the open string tachyon on a non-BPS brane.

B.1 Static solitons

We now show the action, when including some spatial directions, also contain static solitonic solutions which have the correct D-brane tension.

Solitonic solutions to (B.1) can be obtained by taking analytic continuation $t = -ix$ of the $q > 1$ solution (B.7),

$$\sinh \frac{T}{2} = a \cos \frac{x}{2} . \tag{B.11}$$

In the case of D-string, the above solutions correspond to a periodic array of kinks and anti-kinks sitting at $x = (2m + 1)\pi, m \in \mathbf{Z}$. As $a \rightarrow \infty$, the size of the kink becomes infinitely thin and sharply localized.

The mass of the kink (or anti-kink) can be obtained by integration an half period (2π) of x , we find that

$$M = \int_0^{2\pi} dx V(T) \sqrt{1 + T'^2} = 2\pi M_0 . \tag{B.12}$$

Note that this is independent of q . Alternatively one can imagine for (B.1) a single soliton is a configuration with (see e.g., [37, 38])

$$T = \begin{cases} -\infty & x < 0 \\ +\infty & x > 0 \end{cases} \tag{B.13}$$

⁷That effective field theory of the form (B.1) cannot reproduce the precise form of the stress tensor for the full-brane derived from string theory was pointed out earlier in [9] (see also [28]). This appears to suggest that higher derivative terms might be contributing to the effective action.

This again has total energy

$$E = M_0 \int \frac{dT}{\cosh \frac{T}{2}} = 2\pi M_0 \tag{B.14}$$

where we have approximated $\sqrt{1+T'^2} \sim T'$. From (B.12) we see that the deformation from the original D-string configuration at $T = 0$ to the periodic array of kink-anti-kinks does not cost energy, i.e. it corresponds to a marginal deformation. Note the ratio of tensions matches precisely to that between D-string and D0-brane given by string theory. In the superstring theory we have an extra $\sqrt{2}$ in the ratios of tensions due to the scaling mentioned before.

B.2 Coupling to closed string modes

Now we couple the above tachyon system to a set of closed string free fields propagating in the bulk

$$S_{\text{tot}} = \frac{1}{\epsilon} S_0[T(t)] + \int dt j(t) \phi(t, x=0) - \frac{1}{2} \int d^d x dt ((\partial\phi)^2 + m^2 \phi^2) . \tag{B.15}$$

In section two we showed that the couplings of physical closed string modes to the D-brane are essentially given by the one-point function of the unit operator in the bulk, i.e. $\rho(t)$ in (2.5)–(2.7). This motivates us to identify $j(t)$ with $L[T(t)]$. Note that we have made explicit a small parameter $\epsilon \propto g$ in the action and there is no other dependence on ϵ in $L[T]$. Thus the equations of motion become

$$\frac{\partial L}{\partial T(t)} (1 + \epsilon K) = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{T}} (1 + \epsilon K) \right) , \tag{B.16}$$

with

$$K = \int dt' G_{\text{ret}}(t, 0; t', 0) L(T(t')) , \tag{B.17}$$

where G_{ret} is the sum of the retarded propagator for all closed string modes. Multiplying \dot{T} to both sides of (B.16) we find that

$$\frac{d}{dt} (H(1 + \epsilon K)) = -\epsilon L \frac{dK}{dt} , \tag{B.18}$$

where H correspond to the Hamiltonian of S_0 . Note that the energy going into the radiation is given by

$$E_{\text{rad}} = \int dt dt' L(t) \partial_t G_{\text{ret}}(t, 0; t', 0) L(T(t')) . \tag{B.19}$$

When integrating (B.18) over time we get the modified energy conservation law including the part going into the radiation.

C. Brane deformation due to closed string backgrounds

In this appendix we will discuss a very simple example of the problem of finding the deformation of a boundary state due to a closed string deformation of the background.

Let us start with a flat spacetime. Let us consider a D-brane in the 01 directions. Let us now deform the background by deforming the metric by

$$ds^2 = ds_{\text{flat}}^2 + \epsilon(dx_1^2 - dx_3^2)e^{ikx_2+kx_4}, \tag{C.1}$$

where $\epsilon \ll 1$. The deformation grows for large x_4 , but since the brane is localized near x_4 , this is not important for our problem. We view (C.1) simply as the leading term for the expansion of the metric around x_4 .

This deformation will translate into the insertion of the vertex operator

$$\mathcal{O}(z, \bar{z}) = -\frac{1}{\pi\alpha'}\epsilon(\partial X_1\bar{\partial}X_1 - \partial X_3\bar{\partial}X_3)e^{ikX_2+kX_4} \tag{C.2}$$

The operator product expansion as $z \rightarrow \bar{z}$ will be of the form

$$\mathcal{O}(z, \bar{z}) \sim \frac{2i}{z-\bar{z}}V_1 + \dots, \tag{C.3}$$

where the ellipsis indicate operators with other conformal weights and

$$V_1 = \frac{1}{2}\alpha'\epsilon\left(ik\frac{\partial_n X_2}{\pi\alpha'} + k\frac{\partial_n X_4}{\pi\alpha'}\right). \tag{C.4}$$

According to the arguments given in the text the boundary deformation should obey

$$(L_0 - 1)V_B = V_1. \tag{C.5}$$

This motivates us to look for solutions of the form

$$V_B = Y^2(X^0, X^1)\frac{\partial_n X_2}{\pi\alpha'} + Y^4(X^0, X^1)\frac{\partial_n X_4}{\pi\alpha'} \tag{C.6}$$

where (C.5) implies

$$(-\partial_0^2 + \partial_1^2)Y^2 = \frac{1}{2}\epsilon ik, \quad (-\partial_0^2 + \partial_1^2)Y^4 = \frac{1}{2}\epsilon k. \tag{C.7}$$

Note that V_B and V_1 form a “logarithmic” pair [39–41]. (The fact that we get a complex answer in (C.7) is due to the fact that we took a complex metric in (C.1), taking a real metric we get a real answer.)

Let us now compare with the results we would compute using the Born Infeld action. The action can be expanded to lowest order in small fluctuations Y^2, Y^4 about $X^m = 0$ as

$$S = -T \int dx^0 dx^1 \sqrt{-g} = -T \int dx^0 dx^1 \left(1 + \frac{1}{2}(\partial Y^2)^2 + \frac{1}{2}(\partial Y^4)^2 + \frac{1}{2}\delta g_{11}\right). \tag{C.8}$$

The equations of motion for this action become (C.7) once we use (C.1) for the expression for $\delta g_{11} = \epsilon e^{ikY^2+kY^4}$. We see that indeed the general CFT computation agrees with the answer we expect based on the Born Infeld action. Note that since the (C.1) is independent of x^0, x^1 the solutions of (C.7) will grow at infinity. This is in agreement with the expectation that logarithmic divergences in CFT are cancelled by a change in the long distance boundary conditions for the configuration.

It should be noted that if we give the metric deformation some momentum along the brane. Then the operators in the OPE will not generally have dimension one. It is not clear if we need to deform the boundary state in this case since we do not get a logarithmic divergence. From the Born Infeld point of view we would expect that we need to change the boundary state.

This discussion is closely related to the discussions of brane recoil in [42–44].

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